Visco-elastic perfectly plastic model

The visco-elastic perfectly plastic model is a user-defined model allowing for time-dependent behaviour. The model combines isotropic visco-elastic behaviour, consisting of maximum four series of Kelvin-Voigt elements, with the Mohr-Coulomb failure criterion. The model can be schematized as follows:

\[ \Delta \sigma = P (\Delta \epsilon - \Delta \epsilon^p) + \Delta \sigma^{ve} \]

where \( P \) is the visco-elastic material stiffness matrix and
\( \Delta \sigma \) = total stress increment
\( \Delta \sigma^{ve} \) = visco-elastic stress relaxation increment (from Kelvin-Voigt elements)
\( \Delta \epsilon \) = total strain increment
\( \Delta \epsilon^p \) = plastic strain increment (from Mohr-Coulomb failure criterion)

The model parameters can be specified in the Parameters tab sheet, after selecting the eve_MC user-defined soil model.

\[
\begin{align*}
\text{iSet} & = \text{the material set number (can be viewed in Output)} \\
c & = \text{cohesion} \\
\phi & = \text{friction angle} \\
\psi & = \text{dilatancy angle} \\
\text{Tens} & = \text{tensile strength} \\
\text{nKV} & = \text{number of Kelvin-Voigt elements (max. 4)} \\
G_0 & = \text{instantaneous shear modulus} \\
\nu_0 & = \text{instantaneous Poisson's ratio} \\
G_1 & = \text{shear modulus in first Kelvin-Voigt element} \\
\nu_1 & = \text{Poisson's ratio in first Kelvin-Voigt element} \\
\text{Rel. time}_1 & = \text{relaxation time in first Kelvin-Voigt element (determines } \eta_1) \\
G_2 & = \text{shear modulus in second Kelvin-Voigt element} \\
\nu_2 & = \text{Poisson's ratio in second Kelvin-Voigt element} \\
\text{Rel. time}_2 & = \text{relaxation time in second Kelvin-Voigt element (determines } \eta_2) \\
G_3 & = \text{shear modulus in third Kelvin-Voigt element} \\
\nu_3 & = \text{Poisson's ratio in third Kelvin-Voigt element} \\
\text{Rel. time}_3 & = \text{relaxation time in third Kelvin-Voigt element (determines } \eta_3) \\
G_4 & = \text{shear modulus in fourth Kelvin-Voigt element} \\
\nu_4 & = \text{Poisson's ratio in fourth Kelvin-Voigt element} \\
\text{Rel. time}_4 & = \text{relaxation time in fourth Kelvin-Voigt element (determines } \eta_4) \\
\text{Skip init. epsve} & = \text{Skip initialization of visco-elastic strain in Kelvin-Voigt elements}
\end{align*}
\]

The relaxation time is used to determine the viscosity parameter \( \eta \) from the equation:

\[ \text{Rel. time} = \eta / E \quad \text{where } E = 2G(1+\nu) \]
Note that the model considers the ‘normal’ time and NOT the dynamic time!

Note that the initial (instantaneous) strain increment is dominated by the initial stiffness $E_0$, whereas the long-term strain increment is proportional to the sum of the inverse of all stiffnesses:

$$\Delta \varepsilon_0 \sim \Delta \sigma / E_0$$

$$\Delta \varepsilon_\infty \sim \Delta \sigma \left( \frac{1}{E_0} + \sum \frac{1}{E_i} \right)$$

(summation over all Kelvin-Voigt elements)

where $E_i = 2G_i(1+\nu_i)$

The possibility of skipping initialization of visco-elastic strain is particularly relevant for material supposed to undergo stress relaxation after its activation without any change of loading. Otherwise due to the strain initialization process, the Kelvin-Voigt chains also get pre-stressed to the initial stress value such that equilibrium is immediately achieved with the instantaneous stiffness branch right after the activation preventing any stress relaxation without change of loading.
**Example: One-dimensional compression and unloading with creep**

This example is using the Soil Test facility to show the viscous model behaviour in one-dimensional compression (oedometer). Only one Kelvin-Voigt element is used. The model parameters are listed below and the test conditions are shown at the right-hand side:

- **Material set**: Viscous
- **Model**: User-defined

### Parameters

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{oed}$</td>
<td>20000 kN/m²</td>
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</tr>
<tr>
<td>$G_0$</td>
<td>10000 kN/m²</td>
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<tr>
<td>$G_1$</td>
<td>1000 kN/m²</td>
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</tbody>
</table>

**Required:***

- **Vertical stress**: 1000 kN/m²
- **Stress increment**: 1000 kN/m²

**Phases:**

1. Instantaneous loading to -100 kPa.
2. Creep for 1000 days
3. Instantaneous unloading down to -30 kPa
4. Creep for 1000 days
5. Slow reloading to -80 kPa
6. Slow unloading down to -30 kPa

From the results it can be seen that the stiffness during Phase 1 and Phase 3 is equal to:

$$E_{oed,0} = 2G_0 = 20000 \text{ kN/m}^2$$

whereas the stiffness during Phase 5 and Phase 6 is equal to:

$$E_{oed,\infty} = 2G_\infty = 2 \times (1/G_0 + 1/G_1)^{-1} = 2 \times (1/10000 + 1/1000)^{-1} = 1818 \text{ kN/m}^2$$

Moreover, it can be seen that in Phase 2 and Phase 4 the strain ‘creeps’ from the instantaneous strain to the long-term strain as determined by the long-term stiffness.
Figure 1. Stress-strain diagram

Figure 2. Strain-time diagram